Hydroelastic Analysis of a 1900 TEU Container Ship Using Finite Element and Boundary Element Methods

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Abstract
This paper presents a hydroelastic analysis of a 1900 TEU container ship using finite element and boundary elements. The method of analysis is separated into two parts. In the first part, the *in vacuo* dynamic properties of the container ship were obtained by using a standard finite element method. In the second part of the analysis, the ship structure was assumed vibrating in its *in vacuo* modes when it is in contact with fluid, and the pressure distribution on the wetted surface was calculated separately for each mode. The fluid-structure interaction effects were calculated in terms of the generalized added mass terms. The wetted surface of the container ship was idealized by using appropriate boundary elements , referred to as hydrodynamic panels. A higher order panel method (linear distribution) was adopted for the calculations. In a further analysis, the *wet* calculations were repeated using a finite element software (ABAQUS). The *wet* frequencies calculated from the both analysis were compared, and a very good comparison was obtained between the results.

Keyword: Hydroelasticity, boundary element method, finite element method, container ship, vibration.

1. INTRODUCTION

This paper presents the dynamic characteristics (e.g., natural frequencies and mode shapes) of a 1900 TEU container ship in full load and ballast conditions. The method presented is based on a boundary integral equation method together with the method of images in order to impose appropriate boundary condition on the fluid's free surface. The method proposed in this study has already been successfully applied to structures partially filled with or partially submerged in quiescent and flowing fluids (see, for instance, [1] – [4]). In this investigation, it is assumed that the fluid is ideal, i.e., inviscid, incompressible and its motion is irrotational. It is also assumed that the ship hull vibrates in its *in vacuo* eigenmodes when it is in contact with fluid, and that each *in vacuo* mode gives rise to a corresponding surface pressure distribution on the wetted surface of the ship structure. The *in vacuo* dynamic analysis entails the vibration of the ship hull in the absence of any external force and structural damping, and the corresponding dynamic characteristics of the container ship are obtained using a standard finite element software.

At the fluid-structure interface, continuity considerations require that the normal velocity of the fluid is equal to that of the structure. The normal velocities on the wetted surface are expressed in terms of the modal structural displacements. By using a boundary integral equation method the fluid pressure is eliminated from the problem, and that the fluid-structure interaction forces are calculated in terms of the generalized hydrodynamic added mass coefficients.

During this analysis, the wetted surface is idealized by using appropriate boundary elements, referred to as hydrodynamic panels. To assess the influence of the fluid on the dynamic response behavior of the container ship, the *wet* frequencies and associated mode shapes are calculated.

In a further analysis, a standard finite element software (ABAQUS [5]) was used for modeling both the structural and fluid domains. The fluid domain surrounding the ship hull was idealized by using acoustic fluid elements. The cargo loads, ballast weight, etc. applied as inertia mass elements over the cargo area, etc. The *wet* frequencies and mode shapes were calculated by solving the fluid-structure interaction problem. The *wet* frequency values obtained from the boundary element method (BEM) compare very well with those calculated by ABAQUS.

2. MATHEMATICAL MODEL

2.1 Fluid structure interaction problem

The fluid is assumed ideal, i.e., inviscid and incompressible, and its motion is irrotational and there exists a fluid velocity vector, **v**, which can be defined as the gradient of the velocity potential function ϕ as

$$\mathbf{v}(x, y, z, t) = \nabla \phi(x, y, z, t). \tag{1}$$

where ϕ satisfies the Laplace's equation, $\nabla^2 \phi = 0$, throughout the fluid domain.

Before describing the responses of the flexible structure, it is necessary to assign coordinates to the deflections at various degrees of freedom. One particular set of generalized coordinates having the advantage of being unambiguous and easily commended is the principal coordinates of the *dry* structure (see, for example, Ergin *et al.* [6]). For a structure vibrating in an ideal fluid, with frequency ω, the principle coordinate, describing the response of the structure in the *r*th modal vibration, may be expressed by

$$p_r(t) = p_{0r}\, e^{i\omega t} \tag{2}$$

The velocity potential function due to the distortion of the structure in the *r*th *in vacuo* vibrational mode may be written as follows

$$\phi_r(x, y, z, t) = i\,\omega \phi_r(x, y, z)\, p_{0r}\, e^{i\omega t} \qquad r = 1, 2, \ldots, M \tag{3}$$

where M represents the number of modes of interest, and p_{0r} is an unknown amplitude for the *r*th principal coordinate.

On the wetted surface of the vibrating structure the normal fluid velocity must equal to the normal velocity on the structure and this condition for the *r*th modal vibration of the elastic structure submerged in fluid can be expressed as

$$\frac{\partial \phi_r}{\partial \mathrm{n}} = \mathbf{u}_r \cdot \mathbf{n}, \tag{4}$$

where **n** is the unit normal vector on the wetted surface and points into the region of interest. The vector $\mathbf{u}_r$ denotes the displacement response of the structure in the *r*th principal coordinate and it may be written as

$$\mathbf{u}_r(x, y, z, t) = \mathbf{u}_r(x, y, z)\, p_{0r}\, e^{i\omega t} \tag{5}$$

where $\mathbf{u}_r(x, y, z)$ is the *r*th modal displacement vector of the median surface of the structure, and it is obtained from the *in vacuo* analysis.

It is assumed that the elastic structure vibrates at relatively high frequencies so that the effect of surface waves can be neglected. Therefore, the free surface condition (infinite frequency limit condition) (see, for instance, [3]) for the perturbation potential can be approximated by

$$\phi_r = 0, \qquad \text{on the free surface.} \tag{6}$$

The method of images may be used to satisfy this boundary condition. By adding an imaginary boundary region, the condition given by equation (6) at the horizontal surface can be omitted; thus the problem is reduced to a classical Neumann case. The details of the method of images may be found in Ergin and Temarel [4].

2.2 Numerical evaluation of perturbation potential ϕ

The boundary value problem for the perturbation potential, ϕ, may be expressed in the following form:

$$c(\xi)\phi(\xi) = \iint_{S_W} (\phi^*(s,\xi)q(s) - \phi(s)q^*(s,\xi))\,dS \tag{7}$$

where ξ and s denote, respectively, the evaluation and field points on the wetted surface. ϕ^* is the fundamental solution and expressed as follows,

$$\phi^*(s,\xi) = \frac{1}{4\pi r} \tag{8}$$

$q = \partial\phi / \partial\mathbf{n}$ is the flux, and r the distance between the evaluation and field points. The free term $c(\xi)$ is defined as the fraction of $\phi(\xi)$ that lies inside the domain of interest. Moreover, $q^*(s,\xi)$ can be written as

$$q^*(s,\xi) = -(\partial r / \partial\mathbf{n}) / 4\pi r^3 \tag{9}$$

For the solution of equation (7) with boundary condition (4), the wetted surface can be idealized by using boundary elements, referred to as hydrodynamic panels, and the distribution of the potential function and its flux over each hydrodynamic panel may be described in terms of the shape functions and nodal values as

$$\phi_e = \sum_{j=1}^{n_e} N_{ej}\,\phi_{ej}, \qquad q_e = \sum_{j=1}^{n_e} N_{ej}\,q_{ej} \tag{10}$$

Here, n_e represents the number of nodal points assigned to each hydrodynamic panel, and N_{ej} the shape function adopted for the distribution of the potential function. e and j indicates the numbers of the hydrodynamic panels and nodal points, respectively. In the case of a linear distribution adopted in this study, the shape functions for a quadrilateral panel may be expressed as (see Wrobel [7])

$$\begin{aligned} N_{e1} &= ((1-\varsigma)(1-\eta))/4 \\ N_{e2} &= ((1+\varsigma)(1-\eta))/4 \\ N_{e3} &= ((1+\varsigma)(1+\eta))/4 \\ N_{e4} &= ((1-\varsigma)(1+\eta))/4 \end{aligned} \tag{11}$$

After substituting equations (10) and (11) into equation (7) and applying the boundary condition given in equation (4), the unknown potential function values can be determined from the following set of algebraic equations

$$c_k\,\phi_k + \sum_{i=1}^{m}\sum_{j=1}^{n_m}(\phi_{ij}\iint_{\Delta S_i} N_j\,q^*\,dS = \sum_{i=1}^{m}\sum_{j=1}^{n_m}(\mathbf{u}_{ij}\cdot\mathbf{n}_j\iint_{\Delta S_i} N_j\,\phi^*\,dS),\ k = 1, 2, \ldots, m \tag{12}$$

where m denotes the number of nodal points used in the discretization of the structure. ϕ_{ij} and $\mathbf{u}_{ij}$ represent, respectively, the potential and displacement vector for the jth nodal point of the ith hydrodynamic panel.

2.3 Calculation of generalized fluid-structure interaction forces

Using the Bernoulli's equation and neglecting the second order terms, the dynamic fluid pressure on the elastic structure due to the rth modal vibration becomes

$$P_r(x,y,z,t) = -\rho \frac{\partial \phi_r}{\partial t}, \tag{13}$$

where ρ is the fluid density. Substituting equation (3) into (13), the following expression for the pressure is obtained,

$$P_r(x,y,z,t) = -\rho\,\omega^2\,\phi_r(x,y,z)\,p_{0r}\,\mathrm{e}^{i\omega t}. \tag{14}$$

The kth component of the generalized fluid-structure interaction force due to the rth modal *in-vacuo* vibration of the elastic structure can be expressed in terms of the pressure acting on the wetted surface of the structure as

$$\begin{aligned} Z_{kr} &= \iint_{S_W} P_r(x,y,z,t)\,\mathbf{u}_k\,\mathbf{n}\,dS \\ &= p_{0r}\,\mathrm{e}^{\mathrm{i}\omega t} \iint_{S_W} \rho\,\omega^2 \phi_r\,\mathbf{u}_k\,\mathbf{n}\,dS \end{aligned} \tag{15}$$

The generalized added mass, A_{kr} term can be defined as

$$A_{kr} = \rho \iint_{S_W} \phi_r\,\mathbf{u}_k\,\mathbf{n}\,dS, \tag{16}$$

Therefore, the generalized fluid-structure interaction force component, Z_{kr}, can be written as

$$Z_{kr}(t) = A_{kr}\,\omega^2\,p_{0r}\,\mathrm{e}^{i\omega t} = A_{kr}\,\ddot{p}_r(t) \tag{17}$$

2.4 Calculation of wet frequencies and mode shapes

It should be noted that, in the case when a body oscillates in or near a free surface, the hydrodynamic coefficients exhibit frequency dependence in the low frequency region, but show a tendency towards a constant value in the high frequency region. In this study, it is assumed that the structure vibrates in a relatively high frequency region so that the generalized added mass values are constants and evaluated by use of equation (16). Hence the generalized equation of motion for the dynamic fluid – structure interaction system (see, e.g., Ergin *et al.* [6]), assuming free vibrations with no structural damping, is

$$\left[-\omega^2(\mathbf{a}+\mathbf{A})+\mathbf{c}\right]\mathbf{p} = 0, \tag{18}$$

where **a** and **c** denote the generalized structural mass and stiffness matrices, respectively. The matrix **A** represents the infinite frequency generalized added mass coefficients.

Solving the eigenvalue problem, expressed by (18), yields the *wet* frequencies and associated *wet* mode shapes of the structure in contact with fluid. To each *wet* frequency ω_r, there is a corresponding *wet* eigenvector $\mathbf{p}_{0r} = \{p_{r1}, p_{r2}, \ldots, p_{rm}\}$. The corresponding uncoupled *wet* mode shapes for the structure partially and totally in contact with fluid are obtained as

$$\bar{\mathbf{u}}_r(x,y,z) = \{\bar{u}_r, \bar{v}_r, \bar{w}_r\} = \sum_{j=1}^{M} \mathbf{u}_j(x,y,z)\,\mathbf{p}_{rj}, \tag{19}$$

where $\mathbf{u}_j(x,y,z) = \{u_j, v_j, w_j\}$ denote the *in vacuo* mode shapes of the elastic structure and M the number of mode shapes included in the analysis. It should be noted that the fluid – structure interaction forces associated with the inertial effect of the fluid do not have the same spatial distribution as those of the *in vacuo* modal forms. Consequently, this produces hydrodynamic coupling between the *in vacuo* modes of the structur. This coupling effect is introduced into equation (18) through the generalized added mass matrix **A**.

3. NUMERICAL RESULTS

A container ship is adopted for the numerical calculations, and its main particulars are given in Table 1. Two different loading conditions are considered: full load and ballast conditions. The cargo loads are distributed as inertia mass elements over the cargo area inner bottom plating. Ballast weight, heavy fuel oil and other tank weights are also applied as inertia mass elements. The light weight of the ship (LWT) is 9000 tons. The load groups are given in Table 2 for the full load and ballast conditions. The structural finite element model of the container ship is shown in Figure 1.

Table 1. The main particulars of the container ship

Length (L_{bp})	171	m
Breadth (B)	28	m
Depth (D)	16.10	m
Draught (T)	10.00	m
Speed (V)	19.50	knot

The *in vacuo* dynamic characteristics of the container ship were obtained using the ABAQUS finite element software [5]. This produces information on natural frequencies and the normal mode shapes of the *dry* container ship in vacuum. In these calculations, the ship hull was discretized by using 176 800 structural elements. The *in vacuo* dynamic characteristics of the ship structure are scaled to a generalized structural mass of 1 ton m^2.

Table 2. Load groups in full load and ballast conditions (tons).

	Full Load Condition	Ballast Condition
Cargo loads	17150	-
Ballast weight	3021	11250
Heavy Fuel Oil	1886	305
Marine Diesel Oil	165	36.8
Fresh water	206	26.7
Other weights	165	165

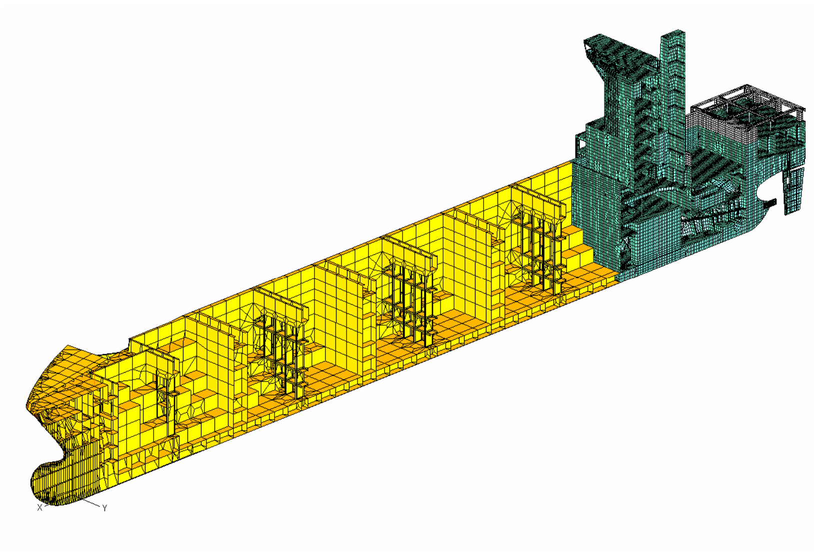


Fig.1 The structural finite element model of the container ship.

It should be noted that the finite element calculations were carried out by the Delta Marine of Turkey. The *dry* natural frequencies of the container ship are given in Tables 3 and 4 for the full load and ballast conditions.

In the *wet* part of the calculations, the fluid is assumed ideal, i.e., inviscid and incompressible, and its motion is irrotational. The wetted surface of the container ship was idealized by using boundary elements, referred to as hydrodynamic panels. 10772 and 6739 hydrodynamic panels were distributed over the wetted surface of the ship hull, respectively, for the full load and ballast conditions. A linear distribution of unknown potentials was assumed over the each hydrodynamic panel. These unknown potentials were calculated using the kinematic boundary conditions imposed at each nodal point. The discritized wetted surface is shown in Figure 2 for the full load condition. The kinematic boundary conditions were obtained from the *in vacuo* finite element calculations. By using Eq.(18), the eigenvalues and eigenvectors of the fluid-structure interaction system were calculated, and the *wet* frequencies are presented in Tables 3 and 4 for the first five global ship hull vibrations together with the *dry* results, respectively, for the full load and ballast conditions. A number of 12 *in vacuo* modes was adopted for the *wet* calculations.

In a further analysis, a standard finite element program (ABAQUS) was adopted for the *wet* calculations. The ship structure and fluid surrounding the ship hull were discretized by finite elements. For the fluid medium, 60 300 acoustic finite elements were employed for the full load condition.

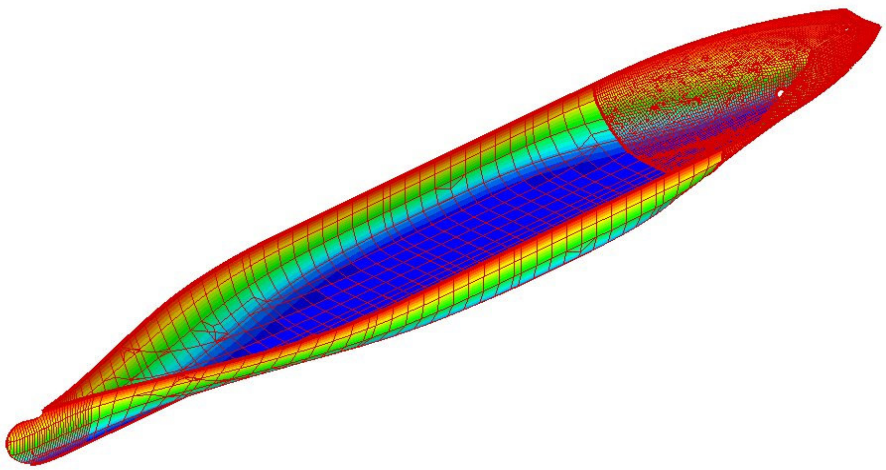


Fig.2 The discretized wetted surface of the container ship for the full load condition.

Table 3. Calculated frequencies for the full load condition.

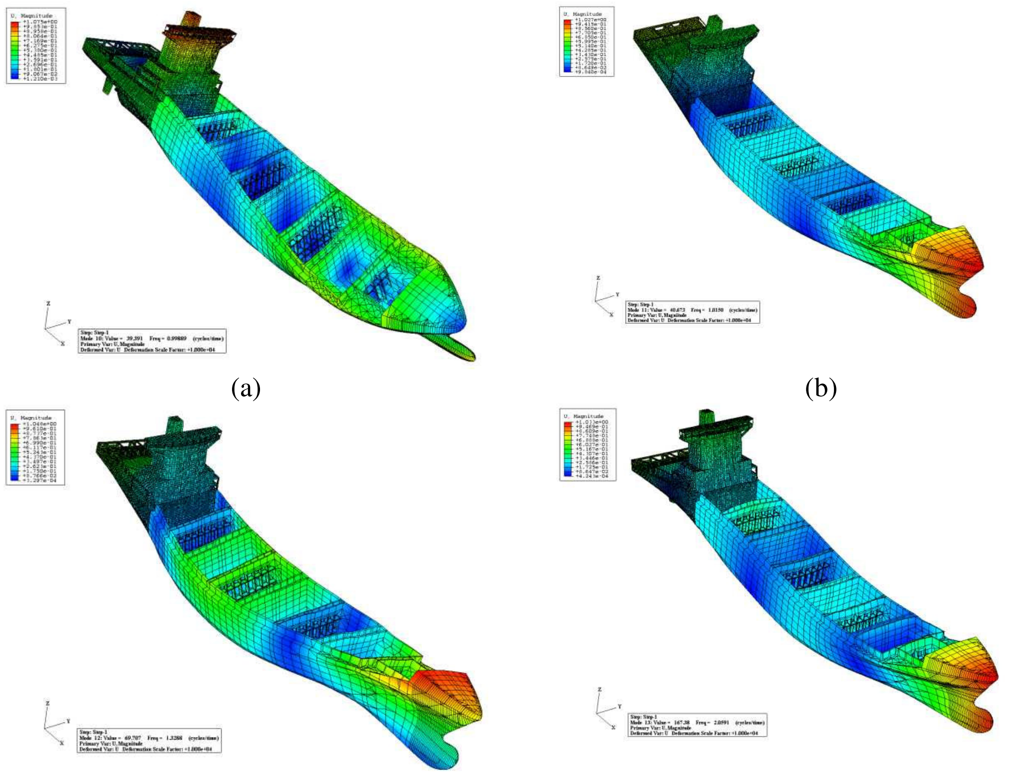
Mode	*In vacuo* FEM	*Wet* FEM	*Wet* BEM	Difference %
1. Torsion	1.119	0.998	0.999	0.1
1. Bending	1.331	1.015	1.041	2.5
Bend.&Tors.	1.515	1.328	1.349	1.6
2. Torsion	2.547	2.325	2.344	0.8
2. Bending	2.676	2.059	2.072	0.6

Table 4. Calculated frequencies for the ballast condition.

Mode	*In vacuo* FEM	*Wet* FEM	*Wet* BEM	Difference %
1. Torsion	1.340	1.221	1.217	0.3
1. Bending	1.608	1.290	1.210	6.2
Bend.&Tors.	1.981	1.787	1.776	0.6
2. Torsion	3.337	2.504	2.386	4.7
2. Bending	3.455	-	3.129	-

It can be seen from Tables 3 and 4 that there are very good agreement between the results obtained from the finite element analysis (ABAQUS) and boundary element analysis proposed in this study. The largest differences obtained are 2.5% and 6.2%, respectively, for the full load and ballast conditions. On the other hand, it can be observed from the tables that the frequencies decrease with increasing area of contact with the fluid. The ship hull has the largest area of contact for the full load condition, and therefore, the lowest frequencies were obtained for this case.

The mode shapes indicated in Tables 3 and 4 are the global vibration modes of the ship hull, and the first four global mode shapes are shown in Figure 3.



(a) (b) (c) (d)

Fig.3 The first four global modes of the container ship: (a) 1st torsion, (b) 1st bending, (c) horizontal bending & torsion, (d) 2nd bending.

CONCLUSIONS

The dynamic characteristics (*wet* frequencies and associated mode shapes) of a container ship for two different load conditions were calculated by an approach based on the boundary integral equation method, and the frequencies obtained were compared with the results of the finite element calculations. A very good comparison was obtained between the predictions of the boundary element and finite element methods. It can be concluded from the results that the boundary integral equation method proposed in this study is suitable for the dynamic analysis of ship hull structures and also for other elastic structures fully or partially in contact with fluid.

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